

# Quantified Modal Logic and the Ontology of Physical Objects

Francesco Belardinelli

January 16, 2007

## Abstract

In this talk I introduce a formal framework to deal with persistence and change of material objects in time. I first consider three ontological theories: perdurantism [6, 7, 17], endurantism [20] and sequentialism [2]; then develop a formal account for these theories by means of semantics for quantified modal logics: Kripke semantics [3, 10], the substantial interpretation [5] and counterpart semantics [1] respectively.

I show that each interpretation soundly formalizes the corresponding ontological account by referring to three features of these semantics: (i) the nature of individuals appearing in the domains of these structures, (ii) the principles on identity sound with respect to each account, (iii) the representation and solution of ontological problems within the logical frameworks. I conclude that Kripke semantics, the substantial interpretation and counterpart semantics are sound formalizations for perdurantism, endurantism and sequentialism respectively.

Finally, I outline some applications of these formalisms to cope with the problem of comparing and reducing ontologies. By results in [4] and [9] I single out the necessary and sufficient conditions according to which an endurantist, a perdurantist and a sequentialist can meaningfully talk to each other.

In the last years there has been a lively debate on the persistence conditions for physical objects in time [13, 14, 18]. The several proposals available in the literature can be roughly divided into three main accounts:

- **Perdurantism** (Hawley [6], Heller [7], Sider [17]), according to which physical objects are extended in time as well as in space, with temporal parts formally analogous to spatial parts.
- **Endurantism** (Wiggins [20]), which maintains the alleged traditional account of persistence<sup>1</sup>: physical objects extend only across space, and persist in time by remaining *wholly present* in every moment in which they exist.
- **Sequentialism** (Chisholm [2], Lewis [11, 12]), which agrees with endurantism in thinking of objects as not extended in time, but differently from endurantism it does not speak of identity of individuals in time. The different temporal stages of an individual are connected by a *counterpart relation* into a unique *ens successivum*.

I develop a formal account for each one of these ontological theses by means of models for quantified modal logic. The key intuition is the same underlying the temporal reading of modal logics: the worlds  $w_1, w_2, \dots$  in a model for quantified modal logic are interpreted as instants in time.

---

<sup>1</sup>In [14] Lowe ascribes endurantism even to Aristotle (*Physics*, Book IV, 10-14).

I introduce three different semantics for a first-order modal language containing only individual variables, no constant nor functors. Formulas are defined as follows:

$$\phi ::= P^n(x_1, \dots, x_n) \mid x = y \mid \neg\phi \mid \phi \rightarrow \phi' \mid \Box\phi \mid \forall x\phi$$

Then I show that each semantics soundly formalizes the corresponding ontological account.

I first consider **Kripke models**, defined as ordered 4-tuple  $\langle W, R, D, I \rangle$  s.t.

- $W$  is the set of instants  $w_1, w_2, \dots$ ;
- $R$  is the precedence relation on  $W$ ;
- $D$  is a function assigning to every instant  $w \in W$ , a set  $D(w)$  of individuals existing in  $w$ ;
- $I$  is an interpretation of our first-order modal language.

I argue that Kripke models soundly formalize endurantism by the following points:

- (i) The individuals in the domains of these structures are *wholly present objects*, as variables are interpreted on the elements belonging to the various  $D(w_1), D(w_2), \dots$ . Moreover, individuals are reidentified with themselves in moving from one to another instant.
- (ii) The principles concerning trans-temporal identity - Leibniz Law  $(x = y) \rightarrow (\phi \rightarrow \phi[x/y])$ , the necessity of identity  $(x = y) \rightarrow \Box(x = y)$  and the necessity of difference  $(x \neq y) \rightarrow \Box(x \neq y)$  - hold both in Kripke models and endurantism.
- (iii) The problems with this ontological account can be represented within the present logical framework. I analyse the puzzles of qualitative and mereological change, as discussed in [19], and the puzzle of coincident but distinct objects[7]. I maintain that the endurantist solution to the first one - consisting in indexing properties to time - can be represented within the Kripke models. On the other hand, the second and third argument can be formally reconstructed, and a contradiction deduced, by the endurantistically valid principles in (ii).

In virtue of all these features common to Kripke models and edurantism, I conclude that the first ones soundly formalize the latter.

Secondly, I present the **substantial models** in [5], defined as Kripke models with in addition a set  $F$  of functions on  $W$  s.t.  $f(w) \in D(w)$ , which intuitively represents the domain of four-dimensional objects stretching across different instants.

The necessity of identity and the necessity of difference - rejected by perdurantists - do not hold in substantial models; this is the case as both in perdurantism and substantial models Leibniz Law is not unrestrictedly valid, but has to be limited to non-modal formulas.

In addition, the perdurantist solution to the puzzle of qualitative change - *via* temporal parts - can be formalized in the substantial models. As to the puzzles of mereological change and of coincident but distinct objects, the contradiction is no longer provable within the present logical framework. Moreover, there exist substantial models formalizing the perdurantist solutions to these puzzles.

Even in this case our semantics for quantified modal logic - the substantial models - is sound with respect to the theses of perdurantism.

Finally, I introduce the **counterpart models**, defined as Kripke models endowed with a counterpart relation  $C_{w,w'} \subseteq D(w) \times D(w')$  for every couple  $\langle w, w' \rangle$  of instants. The peculiarity

of this semantics is that a modal formulas  $\Box\phi$ , with free variables  $x_1, \dots, x_n$ , is true at an instant  $w$  for an assignment  $\sigma$  to the variables iff in every instant  $w'$  accessible from  $w$ , the formula  $\phi$  is true not for  $\sigma(x_1), \dots, \sigma(x_n)$ , but for their counterparts in  $w'$ . This approach was first formulated by Lewis in [11], and note that Lewis' arguments in favour of his Counterpart Theory are motivated by a sequentialist account of persistence in time.

In virtue of the counterpart-theoretic analysis of trans-temporal identity, both sequentialism and counterpart models reject the necessity of identity and the necessity of difference, as an object can have more than one counterpart and can be counterpart to more than one object.

In the end let us consider our puzzles. As it was the case for substantial models, there exist counterpart models formalizing the sequentialist solutions to the puzzles of qualitative and mereological change. We have problems with the puzzle of coincident but distinct objects, as Leibniz Law holds in counterpart models and we can still prove a contradiction. This fact detects a flaw either in the present formal account or in the conceptual apparatus of sequentialism, the subject demands further investigation.

I conclude that our models for quantified modal logic soundly formalize endurantism, perdurantism and sequentialism respectively.

In the final part of my paper I outline some applications of these formalisms to cope with the problem of comparing and reducing ontologies<sup>2</sup>. In [4] Fitting, Kracht and Kutz in [9] introduce translation functions from the validities in counterpart models to the validities in substantial models, which work under precise constraints. According to the *correspondence thesis*, these formal results can be applied to define the constraints by which the truths of sequentialism translate into perdurantistically acceptable statements. I aim at singling out the necessary and sufficient conditions according to which a sequentialist and a perdurantist can meaningfully talk to each other. I extend this formal comparison to endurantism as well.

In the end I stress the role that logic - quantified modal logic in particular - can play in analysing ontological issues as persistence and change of material objects, thus improving our comprehension of the logical consequences of our ontological theses.

## References

- [1] T. Brauner and S. Ghilardi. First-order modal logic. In P. Blackburn, J. van Benthem, and F. Wolter, editors, *Handbook of Modal Logic*. Elsevier, in preparation.
- [2] R. M. Chisholm. *Person and Object*. Open Court, La Salle, IL, 1976.
- [3] G. Corsi. A unified completeness theorem for quantified modal logics. *Journal of Symbolic Logic*, 67:1483–1510, 2002.
- [4] M. Fitting. First order intensional logic. *Annals of pure and applied logic*, 127:173–191, 2004.
- [5] J. W. Garson. Quantification in modal logic. In D. Gabbay and F. Günthner, editors, *Handbook of Philosophical Logic*, pages 267–323. Kluwer Academic Publishers, Dordrecht, 2001.
- [6] K. Hawley. *How things persist*. Clarendon Press, Oxford, 2001.

---

<sup>2</sup>Consider for instance [8, 15, 16] where the authors uphold the equivalence between endurantism and perdurantism.

- [7] M. Heller. *The ontology of physical objects: Four-dimensional hunks of matter*. Cambridge University Press, Cambridge, 1990.
- [8] E. Hirsch. *The Concept of Identity*. Oxford University Press, Oxford, 1982.
- [9] M. Kracht and O. Kutz. The semantics of modal predicate logic ii. modal individuals revisited. In R. Kahle, editor, *Intensionality, Lecture Notes in Logic*, volume 22. ASL, Los Angeles, 2001.
- [10] S. Kripke. Semantical considerations on modal logic. *Acta Philosophica Fennica*, 16:83–94, 1963.
- [11] D. Lewis. Counterpart theory and quantified modal logic. In M. Loux, editor, *The Possible and the Actual*, pages 110–128. Cornell University Press, Ithaca, NY, 1979.
- [12] D. Lewis. *On the Plurality of Worlds*. Blackwell, Oxford, 1986.
- [13] M. J. Loux. *Metaphysics: A contemporary introduction*. Routledge, London, 1998.
- [14] E. J. Lowe. *The Possibility of Metaphysics*. Clarendon Press, Oxford, 1998.
- [15] S. McCall. *A Model of the Universe*. Clarendon Press, Oxford, 1994.
- [16] K. Miller. The metaphysical equivalence of three and four dimensionalism. *Erkenntnis*, 62:91–117, 2004.
- [17] T. Sider. *Four dimensionalism: An ontology of Persistence and Time*. Oxford University Press, Oxford, 2001.
- [18] A. Varzi. Ontologia e metafisica. In F. D’Agostini and N. Vassallo, editors, *Storia della Filosofia Analitica*, pages 81–117. Einaudi, Torino, 2002.
- [19] A. Varzi. Entia successiva. *Rivista di estetica*, 43(1):139–158, 2003.
- [20] D. Wiggins. *Sameness and substance renewed*. Cambridge University Press, Cambridge, 2001.